

COMBINATION TONES IN VIOLINS

Angela Lohri, Sandra Carral and Vasileios Chatziioannou

Institute of Musical Acoustics and International Centre of Harmonics
University of Music and Performing Arts Vienna, Austria
angela.lohri@gmx.ch

ABSTRACT

In this study we investigate the appearance of combination tones in violins. Most modern textbooks emphasise that combination tones occur inside the ear exclusively (intra-aural). In this study this assumption will be subjected to scrutiny based on evidence found in an empirical study, in which combination tones were measured outside the ear (extra-aural).

An experiment was performed in which a violinist played two tones of a particular musical interval simultaneously. This was recorded and subsequently analysed using a Fourier Transformation. In addition to the partial tones of the primary interval the resulting spectrum showed frequencies which correspond to combination tones. Such frequencies may influence the timbre of musical intervals played on the violin.

With a newly devised tone matrix one can compute all potential combination tones that can occur between any pair of partial tones. The detailed analysis of musical intervals by both the frequency spectrum and the tone matrix show in their partial tone structure characteristic mirror and point symmetries.

It is hoped that this research will lead to results relevant for interpreters, composers, violin makers and violin acousticians.

1. INTRODUCTION

In the 1900's the phenomenon "combination tones" became generally known as "psychoacoustic combination tones" or "subjective combination tones". Most modern textbooks emphasise that combination tones occur inside the ear (intra-aural) exclusively. In this study this idea will be discussed on the base of measurements of bowed string instruments. To make progress in the discussion about the real nature of combination tones, this research demands an interdisciplinary approach. Therefore, the authors put great importance on collaboration and comprehension between their disciplines, which are acoustics, mathematics and music.

In the current section the detection of combination tones is discussed. In section 2 the mathematical formulation of the problem is analysed including the derivation of a tone matrix. Section 3 describes a set of experiments carried out on several bowed string instruments and the outcomes of the measurements are discussed in section 4.

1.1 Tartini and Helmholtz

The phenomenon of combination tones was detected by the famous Italian violinist and music pedagogue Giuseppe Tartini in 1714. He discovered that two musical tones played simultaneously, no matter if on one single instrument or on

two different instruments, generated a third tone, in Tartini's words "*terzo suono*". He observed that, lying lower than the played interval, the third tone acted like a bass giving the interval a third dimension, a subtle harmonic context.[1], pp. 13-17. Tartini, not only a genius musician but also a genius thinker, wrote down the detected correlations between mathematics and the third tones in his book *Trattato di musica secondo la vera scienza dell'armonia* ("Treatise on Music according to the True Science of Harmony"). His insights about the principles of the third tone had a significant influence on the development of music theory. Tartini's discovery opened up entirely new possibilities of how sound quality can be understood and realized by practising musicians. *Il terzo suono* served him especially as a tool to control the intonation of intervals and to build entire scales in absolutely precise proportions.

"When I play a double stop on my violin I can physically meet the form of the interval: its physical sign and proof is the third tone which must result. I have thus for myself and for my students the benefit of reliable intonation, and consequently, the benefit of the actual use of the above-mentioned scale, with its precise proportions."¹ [Transl. by Ph. Borer]

The contemporary term for the "third tone" is "difference tone" which is actually only a sub-category of the entire family of combination tones.

In the 19th century, the third tone was studied more systematically. The term "combination tone", introduced by Vieth in 1805, was then generally adopted.

About 150 years after the discovery of combination tones by Tartini, Hermann von Helmholtz published his ground-breaking book *Die Lehre von den Tonempfindungen* ("On the Sensations of Tone") in 1862. Chapter 7 of that book is exclusively dedicated to the phenomenon of combination tones.

Apart from the detection of summation tones, Helmholtz presented an entirely new and revolutionary explanation of combination tones ('distortion theory') and performed experiments where he measured extra-aural combination tones (combination tones that are generated outside and independently of the ear). He called them "objective combination tones" and demonstrated their existence on a polyphonic siren and on a harmonium applying membranes that began to vibrate if the specific frequency occurred.

Helmholtz's new conclusions were accepted universally. Until 1906, inspired by the new theories of Helmholtz, several studies appeared about this second type of combination tones, the "objective combination tones". [2], p.1114. Since that time, research about objective combination tones seems to stagnate. Plomp explains: "By the rapid development of electronics in the

¹Giuseppe Tartini, [1], p. 100: "*Io nel mio violino, dove suonando a doppia corda posso incontrar fisicamente la forma dell'intervallo, di cui è segno fisico dimostrativo il tal terzo suono, che deve risultare, ho il vantaggio per me, e per i miei scolari della sicura intonazione, e in conseguenza dell'uso reale della scala suddetta in precisione di ragioni.*"

first decades of this century, the phenomenon of nonlinear characteristics became so familiar that the existence of combination tones lost much of its enigmatic character.” [2], p.1115.

This may be true as far as the aspects of physics are concerned, but regarding music performance, the significance of combination tones has not been sufficiently investigated.

2. MATHEMATICAL FORMULATION

2.1 Definitions

2.1.1 Nonlinear systems

Combination tones are the product of nonlinear, acoustic transmission systems [3], p.36. The human ear or a string instrument are examples of such a system. We are dealing with a nonlinear system if the response of the system is not equal to the sum of the original signals.

2.1.2 Intra-aural and extra-aural combination tones

For the purpose of clarity, combination tones that are generated inside the ear will be called „intra-aural“, those that occur outside and independently of the ear „extra-aural“.

In contrast to the intra-aural combination tones, **extra-aural combination tones** (ECT) can be measured as vibrations in the air. They follow exclusively physical principles.

Intra-aural combination tones (ICT) on the other hand cannot be measured using a microphone, but have their origin in the human ear. Accordingly, how intense intra-aural combination tones are perceived varies depending on the individual auditory habits of the person.

2.1.3 Mathematical expression

The response of linear system that is being excited by two frequencies f_1 and f_2 can be represented by a linear equation as below.

$$y = (\cos f_1 + \cos f_2) \quad (1)$$

If the system is nonlinear, based on the “distortion theory” of von Helmholtz, we use a function of this type:

$$y = a(\cos f_1 + \cos f_2) + b(\cos f_1 + \cos f_2)^2 + c(\cos f_1 + \cos f_2)^3 + \dots \quad (2)$$

with $a + b + c + \dots = 1$

Because both extra-aural and intra-aural combination tones are generated due to a nonlinear system, they can be described with the same mathematical equations. This is not self-evident considering that a body of a violin is morphologically different from a human ear.

The presence of intra-aural combination tones has been studied extensively by Helmholtz (1863,[6]), Krueger (1908), Stumpf (1910), Husmann (1953,[5]), Plomp (1965,[2]), Zwicker/Feldkeller (1967,[4]) and Smoorenburg (1972), to mention just a few. Their results show that for simple primary tones the average investigator could hear combination tones of the second and third order, what corresponds to the quadratic and cubic distortion of the human ear.

2.1.4 Difference tones and summation tones

The complicated formulas mentioned above (2) are not indispensable to understand the origin of combination tones. At first, it is very important to make the distinction between difference tones and summation tones. Von Helmholtz defined:

“Combination tones are subdivided in two categories. The first one, detected by Sorge and Tartini, which I have called difference tones, are characterized by the fact that their frequencies are equal to the difference between the frequencies of the primary tones. The second category, the summation tones, are detected by myself; their frequencies are equal to the sum of the frequencies of the primary tones.”² [Transl. by A. Lohri]

Summation tones are more difficult to hear. They are subjected to the masking effect of the primary tones, whereas most of the relevant difference tones are not masked because they are lower than the primary tones.

2.2 Tone matrix

Furthermore summation tones and difference tones can occur between any pair of partial tones. To represent systematically all possible combinations between partial tones, a special tone matrix has been devised. Inserting the ratio of a particular musical interval or the frequencies of the interval themselves, the tone matrix computes all potential combination tones that could be generated by this interval (see Fig. 2). This tone matrix shows that the same combination tone can be generated via different partial tone pairs. Inserting the frequency ratio of the primary tones in the yellow boxes at the left, the matrix computes all possible sums (summation tones) and subtractions (difference tones) between pairs of partial tones.

In music theory the representation of intervals in frequency ratios (or in string length proportions) is fundamental. "In Pythagorean tradition, the *quality* expressed by a musical tone corresponds to the *quantity* expressed by a number."³ Moreover, operating with simple ratios, interesting structures can be observed in the matrix. Determined by its ratio, each interval generates characteristic point symmetries (see Fig. 2). The same symmetries are visible in the corresponding spectrum too, but changed from point symmetry into mirror symmetry (see Fig. 1).

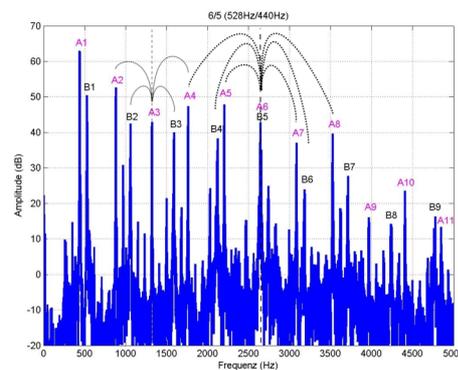


Fig. 1. Spectrum of the third 5/6 played on the violin. The structure of partial tones and combination tones show characteristic mirror symmetries. More details in [7].

²Hermann von Helmholtz, [6] p. 240: “Die Combinationstöne zerfallen in zwei Klassen. Die erste, von Sorge und Tartini entdeckte Klasse, welche ich Differenztöne genannt habe, ist dadurch charakterisiert, dass ihre Schwingungszahlen gleich sind den Differenzen zwischen den Schwingungszahlen der primären Töne. Die zweite Klasse, die Summationstöne, sind von mir entdeckt; ihre Schwingungszahlen sind gleich der Summe der Schwingungszahlen der primären Töne.”

³Philippe Borer, Conversation with A. Lohri, April 2008

70	-64	-58	-52	-46	-40	-34	-28	-22	-16	-10	-4	2
65	-59	-53	-47	-41	-35	-29	-23	-17	-11	-5	1	7
60	-54	-48	-42	-36	-30	-24	-18	-12	-6	0	6	12
55	-49	-43	-37	-31	-25	-19	-13	-7	-1	5	11	17
50	-44	-38	-32	-26	-20	-14	-8	-2	4	10	16	22
45	-39	-33	-27	-21	-15	-9	-3	3	9	15	21	27
40	-34	-28	-22	-16	-10	-4	2	8	14	20	26	32
35	-29	-23	-17	-11	-5	1	7	13	19	25	31	37
30	-24	-18	-12	-6	0	6	12	18	24	30	36	42
25	-19	-13	-7	-1	5	11	17	23	29	35	41	47
20	-14	-8	-2	4	10	16	22	28	34	40	46	52
15	-9	-3	3	9	15	21	27	33	39	45	51	57
10	-4	2	8	14	20	26	32	38	44	50	56	62
5	1	7	13	19	25	31	37	43	49	55	61	67
0	6	12	18	24	30	36	42	48	54	60	66	72
-5	11	17	23	29	35	41	47	53	59	65	71	77
-10	16	22	28	34	40	46	52	58	64	70	76	82
-15	21	27	33	39	45	51	57	63	69	75	81	87
-20	26	32	38	44	50	56	62	68	74	80	86	92
-25	31	37	43	49	55	61	67	73	79	85	91	97
-30	36	42	48	54	60	66	72	78	84	90	96	102
-35	41	47	53	59	65	71	77	83	89	95	101	107

Fig. 2. The example shows the third 5/6. Partial tones are colored in yellow. In the upper quadrant are the difference tones, in the lower the summation tones.

3. MEASUREMENTS

In contrast to intra-aural combination tones, research about extra-aural combination tones has been neglected since the early 20th century. As far as the writer knows, the latest measurements of extra-aural combination tones date from 1906, performed by E. Waetzmann (“Zur Frage nach der Objektivität der Kombinationstöne”, Ann. Phys. 20, 837-845). Due to the author’s presumption that combination tones on the violin exist already outside the ear, in December 2009 a series of measurements have been performed.

3.1.1 Methodology

Two different methods have been applied.

Method 1

In an anechoic chamber, the string player played two notes simultaneously that corresponded to a particular musical interval. The sound was recorded using a microphone (Roga RG-50), positioned near the f holes. A section of approximately two seconds duration was taken from the recorded sound, and its spectrum was calculated using an FFT. To get more information from the measurements we could remove or add following parameters directly on the violin:

- A: with free vibrating strings
- B: with sound post/without sound post

Method 2

Instead of a bow, an electrodynamic mini-shaker (Brüel & Kjaer Type 4810) that produced two simultaneous frequencies excited a violin fixed at its neck and ribs. The contact point of the shaker was placed on the corner of the bridge or on the body next to the bridge. The vibrations were measured using an accelerometer fixed on the body next to the bridge to obtain the vibration of the instrument’s body. To find out where combination tones originate from, we used violins specially prepared for the measurements. Performing these measurements in a violin maker’s studio we were able to change or combine the following parameters:

- A: with free vibrating strings/with damped strings
- B: with sound post/without sound post
- C: without strings and without bridge

4. RESULTS AND DISCUSSION

Sections 4.1-4.5 expose some intermediate results and perspectives of our ongoing research project.

4.1 Measurement results from methods 1 and 2

With both methods various extra-aural combination tones were detected.

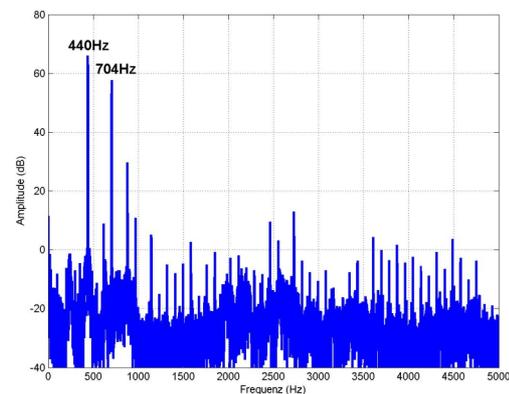


Fig. 3. Excitation of the violin body with 440Hz and 704Hz. A shaker generating two frequencies excited the violin on the bridge (method 2). Numerous combination tones were observed.

To obtain significant results, more measurement series have to be done. Especially the data of method 2 must be confirmed by further measurements. Due to the fact that changes in any part of the violin (see 3.1.1) may cause major differences in the resonance behaviour, it is very difficult to determine the origin of combination tones. Nevertheless, the following intermediate results can be mentioned:

We expect that the sound post increases the nonlinearity in middle and high frequency regions whilst low combination tones are damped out. Another hypothesis is that the wood of the body of the violin has nonlinear properties.

Nonlinearities might appear in violins where two different elastic materials are in contact, but still have certain freedom to move independently [6]. Such transmission systems can easily be found in a violin. Namely the strings, the bridge or the sound post are possible sources of freely vibrating components that could hold potential nonlinear behaviour.

4.2 Violin, viola and violoncello

Extending the experiment, viola and violoncello players were invited as well. To establish a basis for comparison the same intervals were played on all three instruments on the two analogous strings. The data of measurements show that

combination tones appear in all three instruments. The positions of combination tone peaks are determined by the distance (frequency ratio) of the fundamental tones. The amplitudes of combination tones depend on the amplitudes of the partial tones and on the individual resonance behaviour of each instrument. Generally it can be observed that the violoncello tends to generate less extra-aural combination tones than the violin. Regarding the violin, a preliminary test showed that the wood tone (about 200-240Hz) and the air resonance (about 290Hz) might favour the appearance of extra-aural combination tones in these frequency regions.

4.3 A virtual pitch as greatest common divisor

When a musical interval is played on the violin, the sensitive ear can perceive not only combination tones but also a virtual pitch, which corresponds to the greatest common divisor of the two primary tones.

Experiments at the University of Vienna, Department of Musicology, lead us to assume that the appearance of combination tones has a significant influence on the strength of that virtual pitch. Filling in the blanks between partial tones, combination tones complete the series of a grid of equidistant frequencies (see Fig. 4) that suggest the harmonic series of a virtual pitch. This is similar to the „residual tone“ phenomenon.

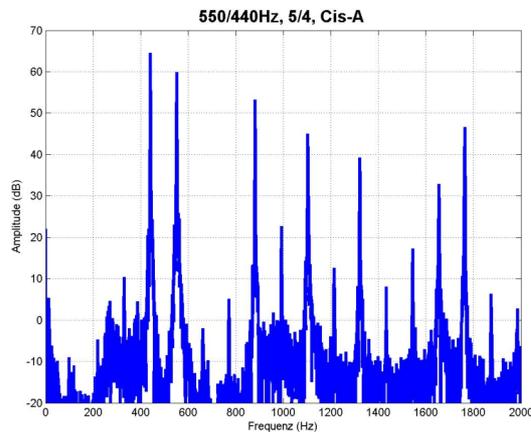


Fig. 4. Spectrum of the interval a'/c-sharp". Partial tones (higher peaks) and combination tones (lower peaks) build a regular pattern. The peaks of combination tones are not chaotically distributed, in contrary, they can appear only in a pattern of equal distance, which is determined by the greatest common divisor (GCD) of the two primary tones. In our example this "virtual pitch" is 110Hz, the greatest common divisor of 440Hz and 550Hz. (Recording: Angela Lohri and Sandra Carral in Dec. 2009)

4.4 Measurements compared with the perception of the listener

Woodhouse points out: "It is never safe to assume that because a particular effect is small in terms of physical measurements, it will not be significant to a skilled performing musician." [8] This also applies the other way round. With the sound processing program "Jaco" it was possible to extract combination tone frequencies from recorded sounds. The aim was to extract a conspicuously audible combination tone that is present as a Fourier component and compare its audibility in the isolated condition with the audibility when embedded in the original sound. It happened that a combination tone, that was clearly

audible in the recording, became much weaker when isolated from the original sound. These circumstances suggest that both mechanisms (ICT and ECT) are active in complex sounds. We may even conclude that in most cases the intra-aural phenomenon has a stronger influence on the loudness of combination tones than the extra-aural phenomenon.

Accordingly, we have to be prudent with statements about the audibility of instrumental combination tones. It is easy to confuse the intra-aural with the extra-aural phenomenon. It is our assumption that what we hear in reality is a mixture of both types (ICT and ECT).

4.5 Perspectives

This study opens up further questions regarding the importance of combination tones in music performance and acoustics of string instruments.

- How important is the effect of extra-aural combination tones in violin playing?
- Are intra-aural combination tones more important in the performance of music than extra-aural combination tones?
- How important is the natural resonance of the instrument regarding the amplitude of particular combination tones?
- How is the combination tones appearance of high quality instruments compared to that of low quality instruments?
- How many orders of combination tones have to be considered to get a mathematical model for extra-aural combination tones generated by two frequencies?

Furthermore, it would be interesting to investigate the existence of extra-aural combination tones in other musical instruments as well.

5. REFERENCES

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