IN SEARCH OF A SOURCE/FILTER MODEL FOR BRASS INSTRUMENTS

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ABSTRACT

Source/filter models are frequently used to model sound production of the vocal apparatus and musical instruments. Beginning in 1968, in an effort to measure a filter characteristic (aka transmission response) of a trombone while it is being played by an expert musician, sound pressure waveforms from the mouthpiece and the bell output were recorded in an anechoic room and then subjected to harmonic spectrum analysis. Output/input ratios of the harmonic amplitudes plotted vs. harmonic frequency then became points on the trombone's filter characteristic. The first such recordings were made on analog 1/4 inch stereo magnetic tape. Recently digital recordings of trombone mouthpiece and anechoic output signals were made that provide a more accurate measurement of the trombone filter characteristic. Results show that the equivalent filter is a high-pass type with a cutoff frequency around 1000 Hz. Whereas the characteristic below cutoff is quite stable, above cutoff it is extremely variable, depending on level. In addition, measurements made using a swept-sine-wave system verify the high-pass characteristic, but they also show a series of resonances whose minima correspond to the harmonic frequencies under performance conditions. For frequencies below cutoff the two types of measurements correspond well, but above cutoff there is a considerable difference. The general effect is that output harmonics above cutoff are greater than would be expected from linear filter theory, and this effect becomes stronger as performance dynamic increases. This nonlinear effect was verified by theory and measurements in the 1990's [1] and early 2000's [2] which showed that nonlinear propagation takes place in the trombone causing a wave steepening effect at high amplitudes, thus increasing the strengths of the upper harmonics.

1. INTRODUCTION

While publications showing input impedance functions of frequency for wind instruments are quite common (e.g., [3] [4]), there have been very few publications showing pressure transfer functions (aka transmission responses). An exception is by Elliott et al. [5], who measured both input impedance and transfer functions for a trumpet and a trombone. They concluded that, despite the exceedingly high pressure levels that can occur in the trombone mouthpiece (greater than 165 dB SPL), "the magnitudes of the (nonlinear) effects are small compared to the overall, linear behavior of the instrument under normal playing conditions". This already had been discussed by Backus and Hundley [6], who concluded that brass systems are linear and that harmonics were generated in the mouthpiece due to a nonlinear variation of the slit resistance of the vibrating lips.

Meanwhile, in 1968 as part of a project to determine a source/filter model for a trombone, I measured the pressure transfer function under performance conditions using 4 different trombonists in an anechoic chamber at the University of Illinois. The idea was to record the mouthpiece and output pressures for tones performed at various pitches and dynamics on separate tracks of an analog tape and then submit them to harmonic analysis. Then, the transfer functions can be estimated by taking ratios between the amplitudes of the corresponding harmonics of the output and input. Thus, for each harmonic k and fundamental frequency f_1, we can define

$$T(kf_1) = P_{out}(kf_1)/P_{in}(kf_1), \qquad (1)$$

which gives the transfer function $T(f)$ in terms of the input pressure P_{in} and the output pressure P_{out} sampled at frequencies kf_1 . Assuming a linear system and constant f_1, $T(f)$ should be independent of dynamic level. However, our measurements, reported in 1969 and 1980, were showing otherwise [7] [8].

1.1 Swept sine measurements

In 1972 I made swept-sine measurements of $T(f)$ in the U of I anechoic chamber for a Holten tenor trombone with microphone positioned 2 m from the bell. The results are shown in Fig. 1.
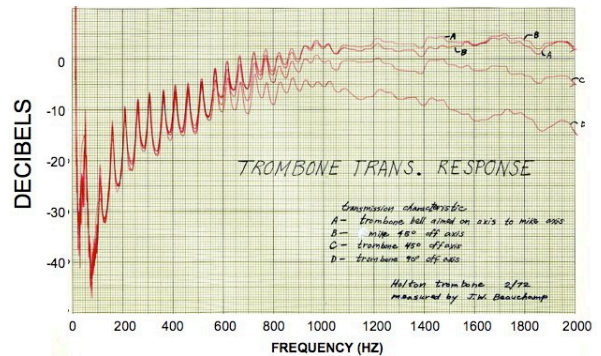

Figure 1. Tenor trombone pressure transfer function (closed position). Four different cases for trombone output pressure measured: A- on-axis; B- on-axis with mic rotated 45°; C- 45° off-axis; D- 90° off-axis.

In 1973 I visited Arthur Benade at Case Institute in Cleveland and together we performed a simultaneous measurement of the transfer and the input impedance functions of my Conn 80A Bb cornet, using the swept-sine/chart recorder method. The graphs, shown in Fig. 2, clearly demonstrate that the local minima of the transfer function curve correspond to the local maxima of the input impedance curve (given by $Z_{in}(f) = P_{in}(f)/U_{in}(f)$, where U_{in} is the mouthpiece particle velocity). It is well known that the local maxima of $Z_{in}(f)$ correspond to the performance frequencies of the instrument, and therefore these frequencies correspond to the local minima of the transfer function.

A comparison of swept-sine and the performance-condition transfer function results based on the 1968 recordings was given at two subsequent talks [9] [10]. However, there was always a little doubt about the accuracy of the performance-condition curves because analog tape has a limited signal-to-noise ratio (approx. 55 dB) as well as significant distortion, so that accurate calculation of the FFT ratio between output and input when the

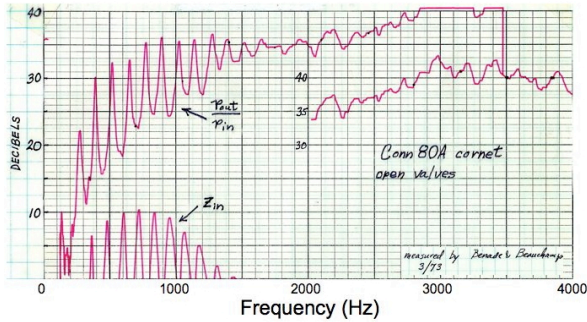upper harmonics of the input are weak (especially for the *pp* case) could be compromised.

Figure 2. Simultaneous measurement of the transfer function (upper curve) and the input impedance (lower curve) for a Bb cornet (open valves). Mic positioned very close to bell.

Therefore, in 2000 I made new direct-to-digital stereo recordings of the mouthpiece and output pressure of a trombone played by Jay Bulen in the University of Iowa anechoic chamber (the U. of Illinois chamber was unfortunately decommisioned in the early 1980s). This allowed much more accurate calculations of performance-condition transfer functions. Fig. 3 shows a block diagram of the measurement system.
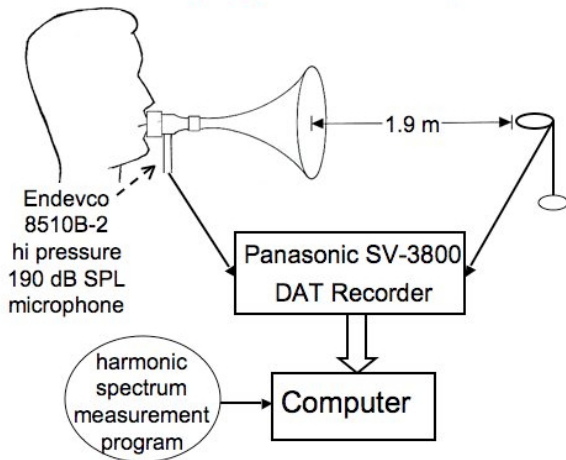

Figure 3. System for measurement of transfer function under performance conditions. A stereo file is stored on the computer with the mouthpiece pressure signal $p_{in}(t)$ as the left channel and output signal $p_{out}(t)$ as the right channel.

2. RESULTS

2.1 Calculation of $T(f)$ under performance conditions

The trombone mouthpiece (input) and on-axis far-field (output) signals (recorded in 2000) were copied to separate monaural files, and a "phase vocoder" program [11] was used to perform harmonic analysis on the signals. The amplitudes of $P_{in}(f_k)$ and $P_{out}(f_k)$, where $f_k = kf_1$ is the harmonic frequency and $f_1 =$ fundamental frequency, were averaged over 2 seconds within the durations of the sounds before computing the transfer function ratio $T(f_k) = P_{out}(f_k)/P_{in}(f_k)$,.

Graphs of $P_{in}(f_k)$, $P_{out}(f_k)$, and $T(f_k)$, where $f_1 = 58$ Hz for the case pitch B^b_1 are shown (converted into decibels[1]) in Figs. 4 – 6 for dynamics *pp*, *mf*, and *ff*, respectively. (Note that the $T(f)$ curves are shown continuous, but this does not imply anything about the filter characteristics between the harmonics as are indicated by the swept-sine measurements.)
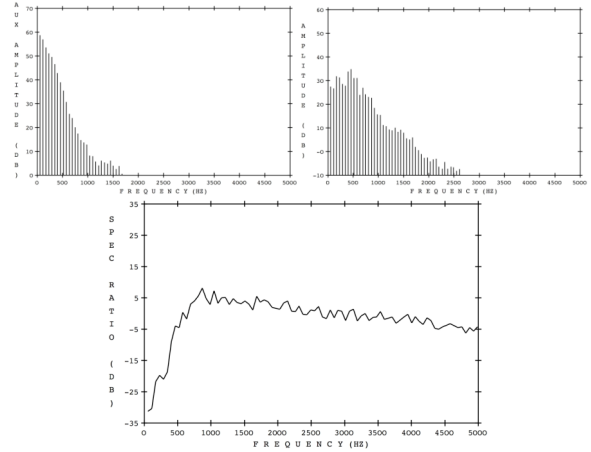

Figure 4. Trombone mouthpiece spectrum (upper left), output spectrum (upper right), and transfer function (lower) for pitch B^b_1 and dynamic *pp*.
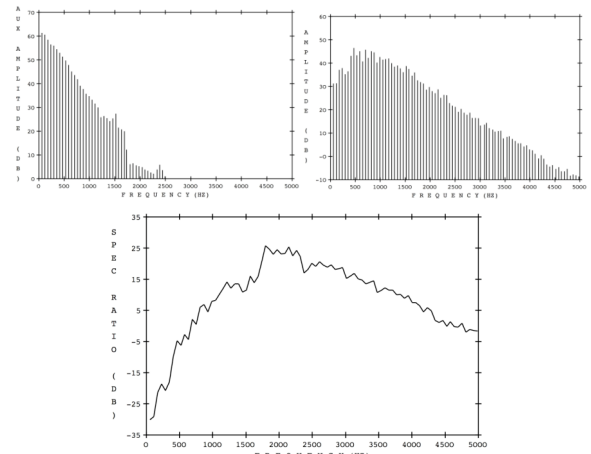

Figure 5. Trombone mouthpiece spectrum (upper left), output spectrum (upper right), and transfer function (lower) for pitch B^b_1 and dynamic *mf*.
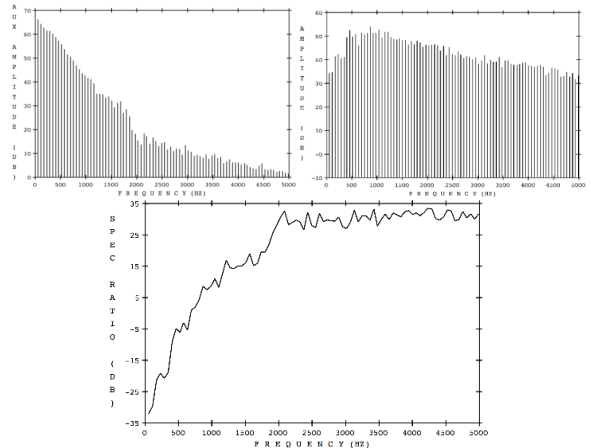

Figure 6. Trombone mouthpiece spectrum (upper left), output spectrum (upper right), and transfer function (lower) for pitch B^b_1 and dynamic *ff*.

Figs. 7–12 show a comparison of $T(f)$ for the *pp*, *mf*, and *ff* cases for pitches B^b_1, B^b_2, F_3, B^b_3, D_4, and F_4, respectively. Two things are obvious from the graphs: First, the curves are nearly the identical for $f < 1000$ Hz. Second, for $f > 1000$ Hz the curves are quite different. In general $T(f)$ is greater as the dynamic level increases. (Note: vertical scales are in decibels and horizontal scales are 0 to 5000 Hz.) It is clear from the graphs that at $f = 5000$ Hz the separation between the *pp* and *ff* cases is on the order of 30 dB.
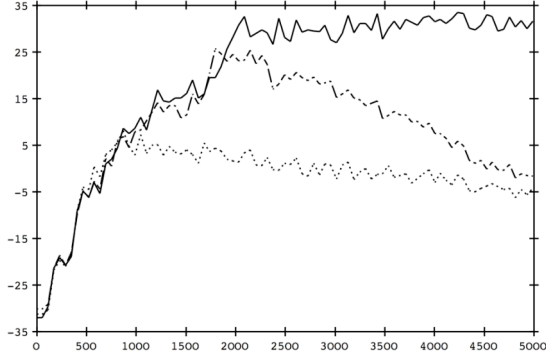
Figure 7. B^b_1 trombone transfer functions compared:
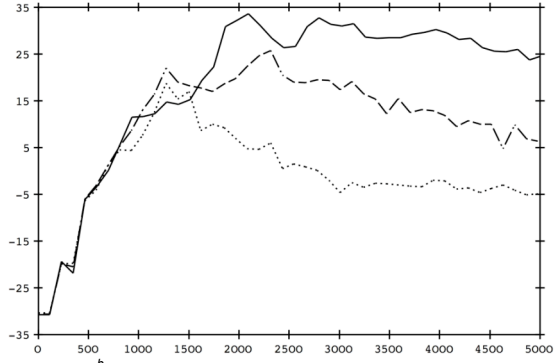*pp* (dotted lower), *mf* (dashed middle), *ff* (solid upper).

Figure 8. B^b_2 trombone transfer functions compared:
pp (dotted lower), *mf* (dashed middle), *ff* (solid upper).

Figure 9. F_3 trombone transfer functions compared:
pp (dotted lower), *mf* (dashed middle), *ff* (solid upper).

Figure 10. B^b_3 trombone transfer functions compared:
pp (dotted lower), *mf* (dashed middle), *ff* (solid upper).

Figure 11. D_4 trombone transfer functions compared:
pp (dotted lower), *mf* (dashed middle), *ff* (solid upper).

Figure 12. F_4 trombone transfer functions compared:
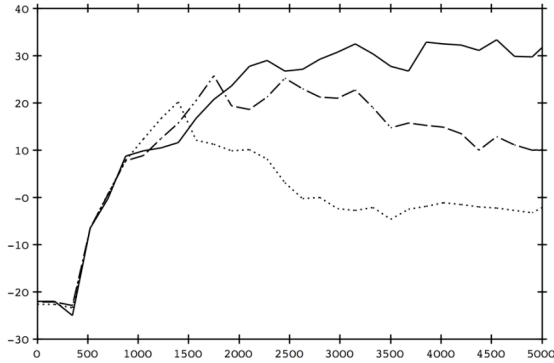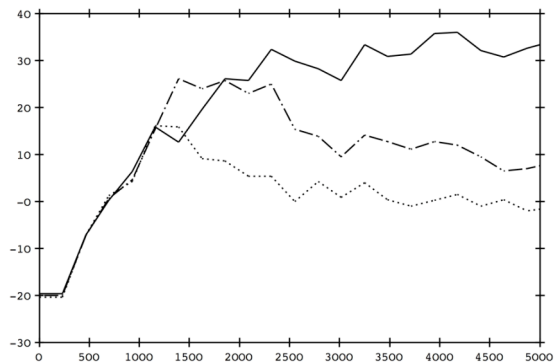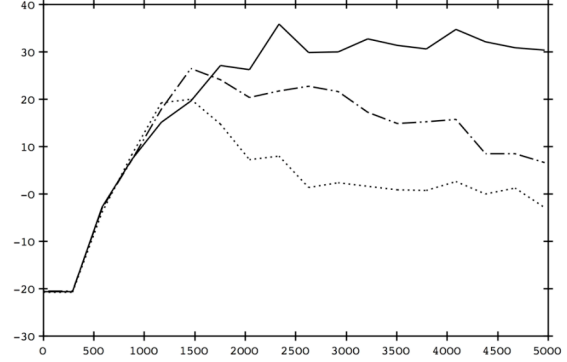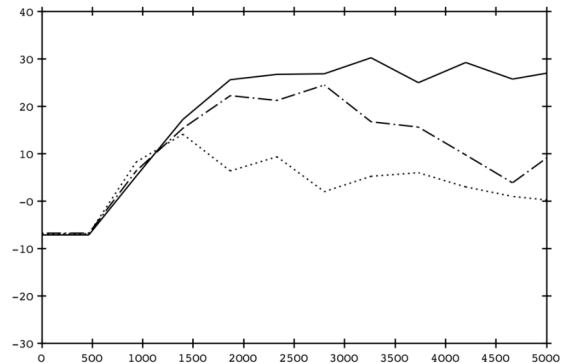*pp* (dotted lower), *mf* (dashed middle), *ff* (solid upper).

2.2 Relationship between $T(f)$ and $Z_{in}(f)$

Assuming a lossless system, we can conclude that output power is equal to input power, i.e., $W_{in} = W_{out}$. For the input in the frequency domain we have

$$W_{in}(f) = \mathrm{Re}\left\{ P_{in}(f)U_{in}(f) \right\} = \mathrm{Re}\left\{ P_{in}^2(f) / Z_{in}(f) \right\}$$
$$= P_{in}^2(f)\mathrm{Re}\left\{ \frac{1}{Z_{in}(f)} \right\} \quad (2)$$

where $U_{in}(f)$ is the input particle velocity in the frequency domain. Note that without loss of generality, under the assumption that $P_{in}(f)$ is real, we can move the real part inside the brackets in the last term to affect only the $1/Z_{in}(f)$ term.

The output power is the average output intensity I_{ave} on a sphere of radius r at which this intensity is measured. Thus, the total output power is

$$W_{out}(f) = 4\pi r^2 I_{ave}(f) = 4\pi r^2 \frac{P_{out}^2(f)}{D(f)Z_o}, \quad (4)$$

where $D(f)$ is the directivity index, Z_o is the characteristic impedance of air, and P_{out} is on-axis pressure output. Equating the input and output powers and solving for $T(f) = P_{out}/P_{in}$ gives

$$T(f) = \frac{P_{out}(f)}{P_{in}(f)} = \sqrt{\frac{Z_o D(f)}{4\pi r^2}\mathrm{Re}\left\{ \frac{1}{Z_{in}(f)} \right\}}. \quad (5)$$

Further, it is approximately true that input impedance functions are real at their local maxima and minima (as indicated by the

zero phase values in [5]). Since the local maxima correspond to the performance frequencies, we can state that

$$T(kf_1) \doteq \sqrt{\frac{Z_o D(kf_1)}{4\pi r^2 Z_{in}(kf_1)}}. \qquad (6)$$

So we see a plausible theoretical explanation of an important aspect of Fig. 2, namely that local maxima and minima of the transfer and the input impedance functions are interchanged. This is most important for frequencies below cutoff, i.e., $f < f_{cut}$, where $f_{cut} \cong 200/d$ and d is the bell diameter [12]. For a trombone, the bell diameter is approximately 0.2 m, and for a trumpet it is approximately 0.1 m, so the cutoff frequencies are about 1000 Hz and 2000 Hz, respectively. We don't have to be concerned about $D(f)$ for $f < f_{cut}$ because it is approximately unity [13]. Above cutoff, however, D increases proportional to f^2 (actually, $D \cong (.01d^2)f^2$). Meanwhile, Z_{in} is dropping from its maximum down to a relatively constant Z_o (see Fig. 2). So, we would expect

$$T(f) \doteq 0.0027 \frac{d}{r} f, \ f > f_{cut} \ . \qquad (7)$$

However, the swept-sine measurements don't verify either Eq. 6 for $f < f_{cut}$ or Eq. 7 for $f > f_{cut}$. Apparently there are internal losses that compensate to produce the actual linear measured result for $f < f_{cut}$. Elliott et al. [5] calculate losses as great as 40 dB for low frequencies and near zero above cutoff, but the question of why $T(f)$ does not increase linearly with frequency for $f > f_{cut}$ remains a puzzle.

2.3 Alignment between swept-sine and performance-condition measurements for $f < f_{cut}$

If we take transfer function values calculated from performance-condition measurements for the trombone and superimpose them on the swept-sine data, how well do they agree? Table 1 shows the data alignment between the two measurements for $f < 1000$ Hz.

The results are very close considering that the trombones were different – a Holton TR602 tenor trombone with a 6 ½ AL mouthpiece was used for the swept-sine measurement of Fig. 1, whereas a Bach 42 B0 tenor trombone with a Stork 5S mouthpiece was used for the most recent performance-condition measurement. Moreover, the measurements were done in different anechoic chambers 28 years apart.

HARMONIC	1	2	3	4	5	6	7	8
FREQUENCY	58	116	174	232	290	348	406	464
DBINPUT	61	61	58	56	56	54	53	51
DBOUTPUT	31	31	37	38	35	36	43	46
DBTRANS	-30	-29	-21	-19	-21	-18	-10	-5
CHART + 3	--	-25	-22	-18	-14	-14	-7	-4
ERROR	--	-4	+1	-1	-7	-3	2	5
HARMONIC	9	10	11	12	13	14	15	16
FREQUENCY	522	580	638	696	754	812	870	928
DBINPUT	50	48	45	44	42	39	38	36
DBOUTPUT	43	45	41	46	42	45	45	40
DBTRANS	-6	-3	-4	2	0	6	7	5
CHART + 3	-5	-4	-3	-3	0	4	4	5
ERROR	-1	+1	-1	+5	0	+2	+3	0

Table 1. For each harmonic of a B^b_1 (58 Hz) *mf* tone, decibel amplitudes (on a relative scale) for input pressure, output pressure, and output-minus-input are given in red (see Fig. 5). Values sampled at the harmonic frequencies from Fig. 1 (upper curve) with 3 added are shown in green. The average magnitude error is approximately 2 dB.

[1] I.e., $T(f)_{dB} = 20 \log_{10}(T(f))$.

3. CONCLUSIONS

Swept-sine measurements of brass pressure transfer functions show a high-pass characteristic with resonance minima corresponding to the harmonic performance frequencies. Measurements of the transfer function under performance conditions closely follow the swept-sine response for harmonics below the cutoff frequency but deviate strongly above cutoff. For example, harmonics above cutoff are roughly 10 to 30 dB stronger for tones played *ff* than for tones played *pp*. A theoretical derivation assuming a linear system, using power conservation, indicates that the transfer function should be inversely proportional to the square root of the impedance. Below cutoff, this relationship is qualitatively verified by swept-sine measurement. However, the detailed relationship is not verified, and the lack of agreement for frequencies above cutoff in the linear case is perplexing.

4. ACKNOWLEDGEMENT

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