EFFECT OF TRANSVERSE ACOUSTIC FLOW ON THE INPUT IMPEDANCE OF RAPIDLY FLARING HORNS

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ABSTRACT

In slowly flaring horns the wave fronts can be considered approximately plane and the impedance can be calculated with the transmission line method (short cones in series). In a more rapidly flaring horn the kinetic energy of transverse flow adds to the local inertance, resulting in an effective increase in length when the horn is located near a pressure node. For low frequencies corrections are available. These might be no longer applicable at higher frequencies when cross-dimensions become comparable to the wavelength, causing resonances in the cross-direction. To investigate this, the pipe radiating in outer space was modelled with a finite difference method. Computer capacity limits the outer space. The outer boundaries must be fully absorbing just as the walls of an anechoic chamber. Applied is Berenger's PML (perfectly matched layer). Presented are results for conical horns, they are compared with earlier published investigations on flanges. The input impedance changes when the largest cross-dimension (outer diameter of flange or diameter of the horn end) becomes comparable to half a wavelength. This effect shifts the position of higher modes in the pipe, influencing the conditions for mode locking, important for ease of playing, dynamic range and sound quality.

1. INTRODUCTION

Most wind instruments are nearly cylindrical or conical. Sometimes they are provided of a horn at the open end, supposedly for increasing the radiation to the space, though the trumpet, definitely louder than a flute, would not need this (see Figure 1).

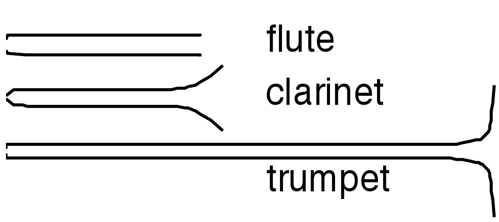

Figure 1: Three wind instruments with and without horns at the radiating open end (*x* and *y* scales are different)

Since a horn is a bore perturbation over a certain length, its influence is dependent on the mode so it may have a function in the tuning of the instrument. The cylindrical flute has a long taper toward the embouchure, which influences tuning, the clarinet ends in a catenoidal horn which compensates for the tuning shifts in the second register due to the speaker hole and the volume of the closed holes [1]. In both cases the bore changes are sufficiently gradual to allow applying the Webster horn equation: wave fronts are approximately plane and the transmission line (TL) method (short cones in series) can be used for calculating the input impedance. Figure 2 shows a basic situation of a cylinder of length L_a joined to a cone of length L_{ba}. The input radius of the cone is a, the output radius b. For low frequencies, assuming the horn short with respect to the wavelength, applying the TL method, the cone can be described as a length correction δ to the cylinder of

$$\delta = (a/b)\left[L_{ba} + (a/b)L_b\right] \tag{1}$$

where in the case of an unflanged open end, $L_b = 0.613b$. (See [2].

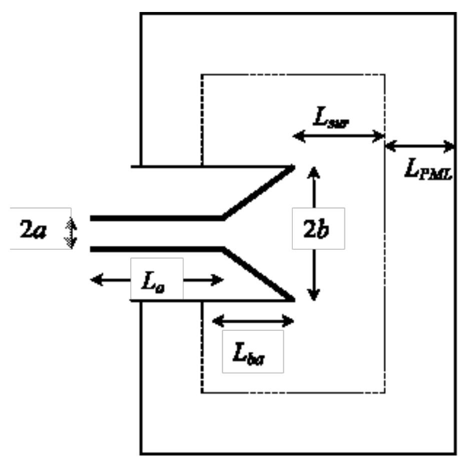

Figure 2. Dimensions of a cylinder with a short cone.

When the flare of the horn is large there will be cross flow which demands kinetic energy. In that case we may apply a fit formula for an extra series impedance to be added to the input impedance of the conical tube piece [2]

$$Z_{\text{extra}} = \frac{j\rho c}{\pi a^2}\frac{a}{b}\,kL_{ba}\left[\sqrt[1.6]{1+\left(\frac{0.821(b-a)}{L_{ba}}\right)^{1.6}}-1\right] \tag{2}$$

The correction becomes important for $(b-a)/L_{ba} > 0.1$. For practical purposes this expression can be used up to $L_{ba} = 0$ (flanges). This may fail to be true at higher frequencies where resonances may occur in the transverse direction. Investigating this was the aim of the present study.

An indication of what may happen can be seen from a previous study of the end correction of a flange. Here it was found that resonances across the flange change the end correction irregularly. [3]. Results are given in Figure 2 for a circular flange with $b = 5a$. As can be observed, a peak in δ/a occurs at $ka \approx 0.3$, or $kb \approx 1.5 \approx \pi/2$, approximately a quarter-wavelength.

This can be expected also to happen in a short horn at the end of a trumpet, which resembles a flange. This may influence the positions of higher overtones.

Calculations were done with the TL method without (called 1D) and with (called 3D) the cross-flow correction, and with the finite difference method (FDM). Results were experimentally validated in the Laboratoire d'Acoustique de l'Université du Maine in Le Mans, France.

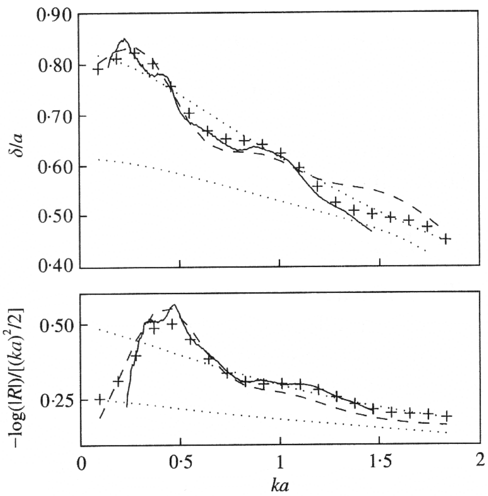

Figure 3. End correction for a flange (b=5a), divided by the pipe radius as a function of ka; top: real part; bottom: imaginary part/(ka). Dotted lines: theory for infinite and zero flange; ++: calculation with boundary element method; drawn line: measurements, dashed line: fit formula. [2]

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2. FDM CALCULATIONS

The Helmholtz equation describes the sound field in the pipe and the open space it radiates into. It was modelled using the finite difference method (FDM). For example, the discretisation Helmholtz equation in two-dimensional Cartesion space is (see Figure 4)

$$\sum_{1}^{4} \frac{p_n - p_0}{h_n} + k^2 p_0 = 0, \tag{3}$$

where $h_n = h$. The points are fitted to the curved walls of the pipe. Since the pipes are rotationally symmetric, cylindrical co-ordinates are used to make the field two-dimensional, saving on computer capacity [2]. Eq(3) is subsequently modified.

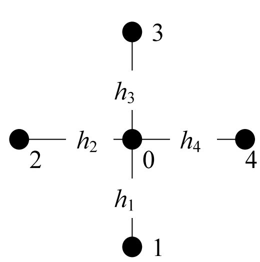

Figure 4. Computational module for FDM calculations

To save on computer capacity the outer space must be bounded in some way. To ensure a smooth transfer in the first outer layer of thickness L_{sur} (for example 10 points) the equation is kept the same. Then follows a layer of thickness L_{PML} in which Berenger's PML (perfectly matched layer) technique was used. In this layer absorption gradually increases, mimicking the absorbing walls of an anechoic chamber. There are various ways to implement this, but it appeared most convenient to make the grid spacing progressively complex to the outside [4, 5, 6]. For example, if the PML absorbing layer is extending to the right the grid dimensions become

$$h_4 = h\left(1 - 5i(m/m_{\max})^3\right) \tag{4}$$

where m is the layer number and $m_{\max}$ the thickness of the absorbing layer (for example 10).

The limited accuracy due to the discretization was improved by plotting the results against the reciprocal of the number of points and extrapolating to zero, yielding an estimate for an infinite number of points.

3. RESULTS

Investigated were various horns attached to a cylindrical pipe. As an introductory example we show some results for a conical pipe of the dimensions

$$[L_a \;\; a \;\; L_{ba} \;\; b \;\; L_{\mathrm{sur}} \;\; L_{\mathrm{PML}}] = [1 \;\; 3 \;\; 22 \;\; 12 \;\; 15 \;\; 30], \tag{5}$$

for ka=1.2 (appr. 6600 Hz). In Figure 5 and 6 the fieldlines of the pressure are plotted in 2D and 3D. The presence of waves in the transverse direction is clearly visible. It can be seen that the pressure smoothly goes to zero toward the boundaries, no unwanted waves due to reflections are visible. From these calculations, the impedance can be obtained as well as a length correction to the cylinder preceeding the cone.

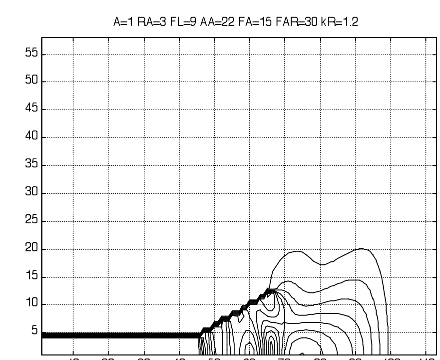

Figure 5. Two-dimensional plot of the lines of equal pressure in a cylinder-cone combination

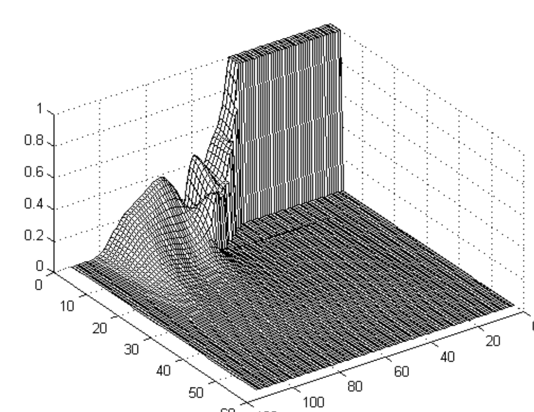

Figure 6. Three-dimensional plot of pressure field in a cyliner-cone pipe combination. Cone to the left of the cylinder.

Measurements were carried out on horns made in the workshop attached to cylindrical tube pieces, in the anechoid chamber of the Le Mans laboratory. For these pipe shapes TL and FDM calculations were carried out. Results were compared.

Since the dimensions are not exactly comparable, some differences remain.

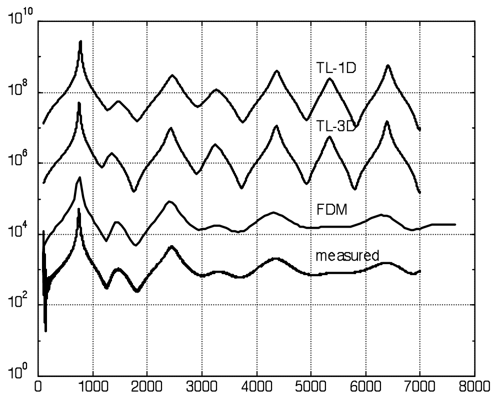

Figure 7 Absolue value of input impedance as a function of frequency for a cylinder with a long cone, determined in four different ways (vertically shifted)

3.1 Cylinder with a long conical horn

First investigated was a cylinder/cone combination (see Figure 8) with the following dimensions (in mm)

$$[L_a \;\; a \;\; L_{ba} \;\; b] = [88 \;\; 9.9 \;\; 75 \;\; 39.5] \qquad (6)$$

Because of its discrete nature the dimensions of the FDM shape had to be slightly different

$$[L_a \;\; a \;\; L_{ba} \;\; b \;\; L_{\text{sur}} \;\; L_{\text{PML}}] = M[18 \;\; 2 \;\; 15 \;\; 8 \;\; 10 \;\; 20], \qquad (7)$$

where $M \geq 2$. M is a multiplier increasing the number of pointrs and consequently the accuracy; it is bounded by the computer capacity, mostly $M \leq 8$. The accuracy is increased by extrapolation to infinity, by plotting the results versus the reciprocal of M and extrapolating to zero.

Measurements on the various pipe combinations were performed at a temperature of 21°C. So in all calculations the velocity of sound at that temperature was assumed, viz. 343 m/s. FDM calculations were performed for ka = 0.02:0.01:1.4, corresponding to a frequency range between 110 and 7700 Hz. Impedances obtained in the four ways described above are plotted in Figure 7. The relative positions of the peaks are given in Figure 8. The exit diameter of the horn, 80 mm, corresponds to a wavelength at 4300 Hz, the frequency where the peaks predicted by the TL methods appear to flatten out according to both measurement and FDM.

The cone can be considered as a length correction to the cylinder, this is plotted in Figure 9. Value at zero (extrapolated) = 2.15. Calculated with eq (4) gives 2.02. The value in publication [2] is 2.33.

Table 1. Position of first resonance peak for a cylinder with a long cone obtained with various methods

Method	TL-1D	TL"3D"	FDM	Measured
1st peak, Hz	766	746	744	748
Difference	+ 2.4 %	– 0.3 %	– 0.5 %	

In Table 1 gives he values of the first peak of the resonance spectrum obtained with the various methods. The value of the 1D-TL calculation is higher, due to neglect of the transverse flow energy in the horn.

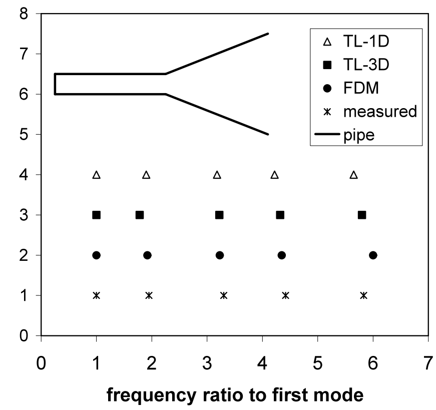

Figure 8. Relative positions of modes in a cylinder with a long cone

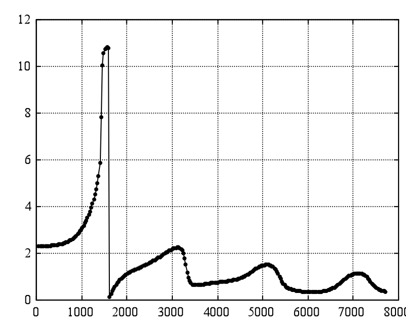

Figure 9. Real part of the end correction coefficient δ/a to the cylinder for the long cone, obtained by FDM.

By way of calibration, a cylinder without horn was investigated. The difference with the measured value of the first maximum was – 0.4% for the TL method, and + 0.3% for the FDM results (probably due to dimensional and temperature uncertainties). It may be concluded that the best forecaster of the impedance at low frequencies is the TL-3D.

3.2 Cylinder with a short conical horn

A cylinder with a short conical horn is sketched in Figuie 11.. The dimensions in mm are:

$$[L_a \;\; a \;\; L_{ba} \;\; b] = [89.8 \;\; 10 \;\; 12 \;\; 19.9] \qquad (8)$$

and those in the FDM model

$$[L_a \;\; a \;\; L_{ba} \;\; b \;\; L_{\text{sur}} \;\; L_{\text{PML}}] = M[18 \;\; 2 \;\; 2 \;\; 4 \;\; 10 \;\; 20], \qquad (9)$$

Results are given in Figures 10 and 11 and in Table 2. Impedance calculations and measurements are plotted in Figure 10. Figure 11 gives relative positions of the peaks. Modes are approximately odd harmonincs. At 6 kHz the peaks flatten out, which does not show up in the TL calculations, but only in both FDM and measurements. Table 2 gives the positions of the first impedance peaks. As can be seen the first peak shifts when the 3D correction is added to the TL method, but the relative position of the peaks do not change much (see Figure 11).

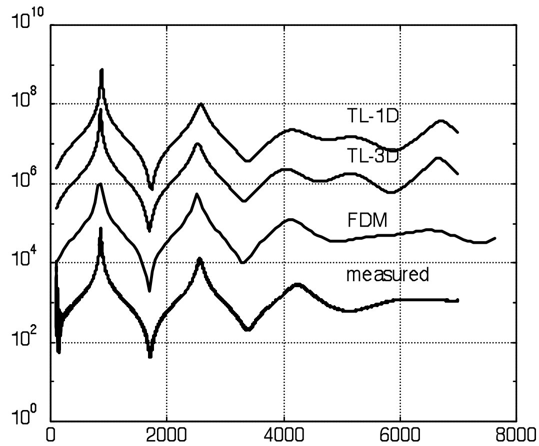

Figure 10. Absolute value of imput impedance of a cylinder with a short cone

Table 2. First resonance peak for a cylinder with a short conical horn

Method	TL-1D	TL-3D	FDM	Measured
1st peak, Hz	865	850	855	859
Difference	+ 0.7%	– 1 %	– 0.5%	

An end correction to the cylinder can be calculated (Figure 12). As can be seen the behaviour is only varying slightly below 5 kHz. Above that frequency the length correction suddenly decreases.

4. DISCUSSION

The position of the first resonance peak of the impedance spectrum is best predicted by TL-3D. The value found with TL-1D can be too high. The short conical horn can be described as a length correction, in a way comparable to that of a flange, provided the frequency is not too high. At higher frequencies the FDM results correspond much better with the measurements than the TL methods. TL methods are fast and conveniently applied. The FDM is cumbersome, a program must be written, there are limitations to the choice of the dimensions.

5. CONCLUSIONS

Since the investigations are still in progress, conclusions are preliminary. For calculating the input impedance of an arbitrary pipe, we conclude that

1. The well-known 1D-TL method is useful for pipes flaring not too much.
2. For horns flaring more than 10% a correction for transverse flow, leading to the so-called 3D-TL method is useful for low frequencies.
3. For higher frequencies, where cross-dimensions are no longer small with respect to the wavelength, the FDM gives better results. This method, however, is cumbersome.
4. Berenger's PML absorbing boundary can be implemented easily in a FDM calculation scheme.

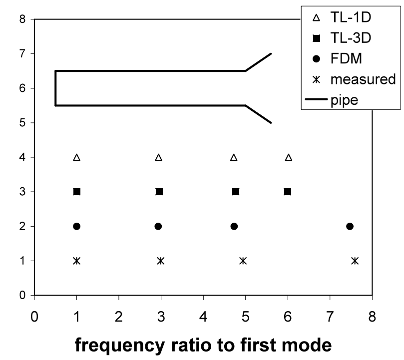

Figure 11. Relative positions of modes in a cylinder with a short cone

6. ACKOWLEDGEMENTS

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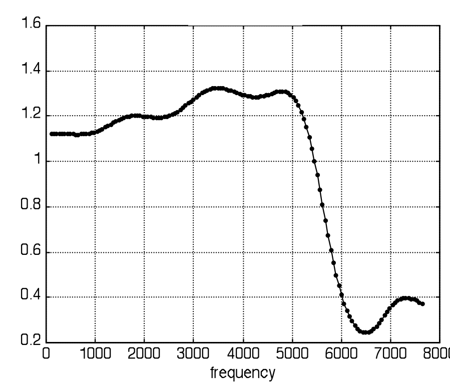

Figure 12. Real part of the end correction coefficient δ/a to the cylinder for the short cone, obtained by FDM.

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