

# THE USE OF VOLTERRA SERIES FOR SIMULATING THE NONLINEAR BEHAVIOUR OF MUSICAL INSTRUMENTS

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## ABSTRACT

The measurement and emulation of audio systems (devices, environments and sound boxes) have been walked in these years. The most-used methods to obtain information about an audio system are those based on measuring its impulse response (IR). Once the IR has been caught it is possible to recreate, by the use of linear convolution, the output signal that the audio system will generate when it is physically driven by any input signal. This method gives great results if the system is linear and time-invariant (environments behaviour is much linear and therefore its reverberant effect can be faithfully recreated using IRs) but not satisfactory in other cases, such as the emulation of tube preamps (mainly nonlinear) and musical instruments. Since the musical instruments cannot be considered completely linear, their musical performance might be analysed properly considering also their nonlinear behaviour.

By using Volterra series it is possible to represent the input-output relationship of nonlinear systems. This mathematical theory uses a set of impulse responses to describe the system and not only one as before. By an enhanced impulse response measurement method it is possible to obtain this set of impulses and then with Volterra series it would be possible to have the output of the audio system driven by any input. A special numerical tool has been developed to recreate the system behaviour by using this method. Satisfactory results have been obtained in comparison with the traditional linear convolution based approach.

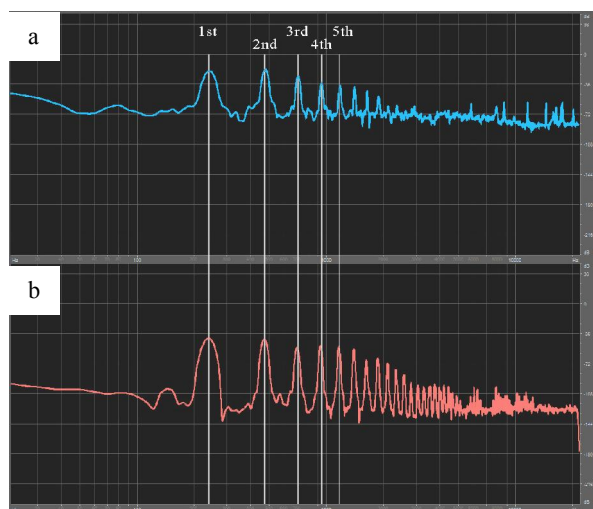


Figure 1: Fundamental and harmonics in an A#3 (233.1 Hz) played by a trombone (a) and a guitar (b).

## 1. INTRODUCTION

Any complex tone "can be described as a combination of many simple periodic waves (i.e., sine waves) or partials, each with its own frequency of vibration, amplitude, and phase." [1]. A harmonic (or a harmonic partial) is any of a set of partials that are whole number multiples of a common fundamental frequency [2]. This set includes the fundamental, which is a whole number multiple of itself (1 times itself). The relative amplitudes of the various harmonics primarily determine the timbre of different instruments (see Figure 1). The ratio of these harmonics (and therefore the timbre) generally varies during the execution of a note if it is played with different dynamics: that's one reason why raising the volume of a pianissimo performance is not the same as listening to it if played fortissimo (see Figure 2). Thinking as the instrument as if it were a black-box (i.e. a system) these behaviours let us understand that it has the key features to be classified as nonlinear.

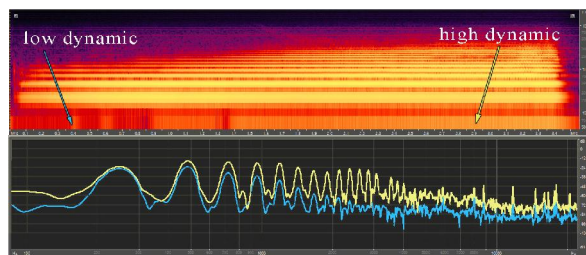


Figure 2: Changes of harmonics ratio while playing the same note on trombone (more harmonics when played louder)

Harmonic order	Frequency (Hz)	Nearest note in equal temperament
1	233.1	A#3 (233.1 Hz)
2	466.2	A#4 (466.2 Hz)
3	699.3	F4 (698.5 Hz)
4	932.4	A#5 (932.3 Hz)
5	1165.5	D5 (1174.7 Hz)

Table 1: Frequency to note relation table

## 2. NONLINEAR SYSTEMS

With a more musical than mathematical approach a nonlinear system can be described as a system that if driven by a pure tone exhibits harmonics with ratios that are correlated with the





performance and a virtualization of the “anechoic” version of that performance has been taken. Nonlinearities of the flute even if well matched by the 5 kernels rendering (Figure 10-b) have little effects on the ESS wave (Figure 11); therefore the emulated performances with and without harmonic distortions will not be much different (Figure 12).



Figure 12: Performances comparison

## 5. CONCLUSIONS

It has been seen that the measurement of harmonic distortion of nonlinear systems with ESS and the following emulation using the diagonal Volterra kernels provides better results than a single IR convolution approach (Figure 8). If the core of the system resides in its low order harmonic distortions the progress of that kind of virtualization is revealed to be evident instead if the system doesn't have sharp harmonic distortions linear convolution is able to provide comparable results (Figure 11 and Figure 12).

As known in nonlinear system harmonics ratios may depend on the volume of the stimulus. To improve the emulation on the whole dynamic of a signal it would be necessary to measure the system with different amplitudes of ESS, collecting the different sets of kernels and based for example on the actual RMS value of a suited length buffered input chose on the fly which set of kernels to use.

## 6. REFERENCES

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