# THE USE OF VOLTERRA SERIES FOR SIMULATING THE NONLINEAR BEHAVIOUR OF MUSICAL INSTRUMENTS

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## ABSTRACT

The measurement and emulation of audio systems (devices, environments and sound boxes) have been walked in these years. The most-used methods to obtain information about an audio system are those based on measuring its impulse response (IR). Once the IR has been caught it is possible to recreate, by the use of linear convolution, the output signal that the audio system will generate when it is physically driven by any input signal. This method gives great results if the system is linear and time-invariant (environments behaviour is much linear and therefore its reverberant effect can be faithfully recreated using IRs) but not satisfactory in other cases, such as the emulation of tube preamps (mainly nonlinear) and musical instruments. Since the musical instruments cannot be considered completely linear, their musical performance might be analysed properly considering also their nonlinear behaviour.

By using Volterra series it is possible to represent the input-output relationship of nonlinear systems. This mathematical theory uses a set of impulse responses to describe the system and not only one as before. By an enhanced impulse response measurement method it is possible to obtain this set of impulses and then with Volterra series it would be possible to have the output of the audio system driven by any input. A special numerical tool has been developed to recreate the system behaviour by using this method. Satisfactory results have been obtained in comparison with the traditional linear convolution based approach.

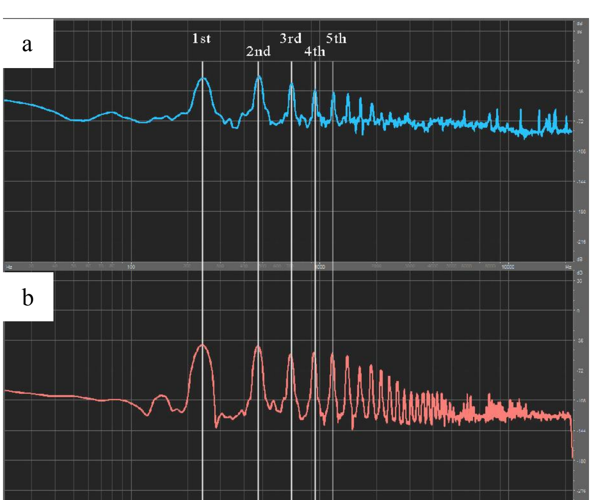



Figure 1: Fundamental and harmonics in an A#3 (233.1 Hz) played by a trombone (a) and a guitar (b).

## 1. INTRODUCTION

Any complex tone "can be described as a combination of many simple periodic waves (i.e., sine waves) or partials, each with its own frequency of vibration, amplitude, and phase." [1]. A harmonic (or a harmonic partial) is any of a set of partials that are whole number multiples of a common fundamental frequency [2]. This set includes the fundamental, which is a whole number multiple of itself (1 times itself). The relative amplitudes of the various harmonics primarily determine the timbre of different instruments (see Figure 1). The ratio of these harmonics (and therefore the timbre) generally varies during the execution of a note if it is played with different dynamics: that's one reason why raising the volume of a pianissimo performance is not the same as listening to it if played fortissimo (see Figure 2). Thinking as the instrument as if it were a black-box (i.e. a system) these behaviours let us understand that it has the key features to be classified as nonlinear.

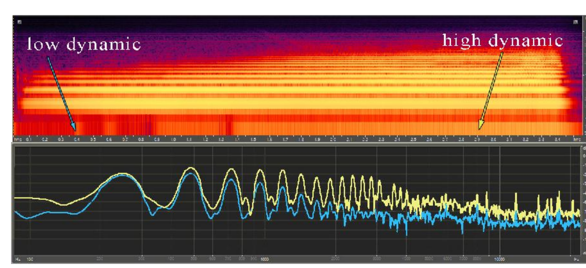



Figure 2: Changes of harmonics ratio while playing the same note on trombone (more harmonics when played louder)

| Harmonic order | Frequency (Hz) | Nearest note in equal temperament |
|---|---|---|
| 1 | 233.1 | A#3 (233.1 Hz) |
| 2 | 466.2 | A#4 (466.2 Hz) |
| 3 | 699.3 | F4 (698.5 Hz) |
| 4 | 932.4 | A#5 (932.3 Hz) |
| 5 | 1165.5 | D5 (1174.7 Hz) |

Table 1: Frequency to note relation table

## 2. NONLINEAR SYSTEMS

With a more musical than mathematical approach a nonlinear system can be described as a system that if driven by a pure tone exhibits harmonics with ratios that are correlated with the volume of the stimulus. A way to formally represent this input-output relation has been developed by Vito Volterra (1860-1940) in 1887.

### 2.1 Volterra series

$$y(t) = k_0 + \sum_{n=1}^{\infty} \frac{1}{n!} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} k_n(t_1, t_2, \ldots, t_n) x(t-t_1) x(t-t_2) \cdots x(t-t_n) dt_1 dt_2 \cdots dt_n \quad (1)$$

Equation (1) represents the input-output relation of a continuous time-invariant system with memory using Volterra series. The term $k_n$ is called *n*-th order Volterra kernel and can be seen as a higher order impulse response of the system. If only the *1*-st kernel differs from zero, the Volterra series reduce to the input-output relation of a time-invariant linear system with memory (i.e. the output is the convolution between input and the impulse response of the system). Imposing that nonlinear part is purely algebraic and that the memory effects reside only in the linear part of the system, eq. (1) can be rewritten as (2) where the terms 1/n! have been embedded in the respective $k_n$:

$$y(t) = k_0 + \sum_{n=1}^{\infty} k_n(t) * x^n(t) \quad (2)$$

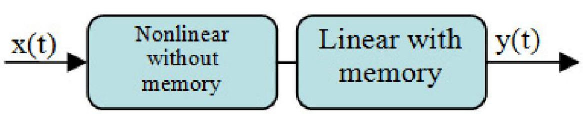



Figure 3: Hammerstein model (diagonal Volterra kernels)

### 2.2 ESS (Exponential Sine Sweep) and harmonic distortions

A new method for measuring the IR has been recently developed [3]. It is based on an exponential sweep of frequencies (ESS) that could collect information even on harmonic distortions generated by nonlinear systems [3]. This information is related to the kernels of (2) and they can be obtained by using the following frequency domain formulas [4]:

$$\begin{cases} K_1 = \dfrac{H_1 + 3H_3 + 5H_5}{\alpha} \\ K_2 = \dfrac{2jH_2 + 8jH_4}{\alpha^2} \\ K_3 = \dfrac{-4H_3 - 20H_5}{\alpha^3} \\ K_4 = \dfrac{-8jH_4}{\alpha^4} \\ K_5 = \dfrac{16H_5}{\alpha^5} \end{cases} \quad (3)$$

where $K_n$ represent the Fourier transform of the *n*-th time domain diagonal Volterra kernel; $H_n$ are the Fourier transform of the *n*-th harmonic responses of the systems (Figure 4); *j* is the 90° phase shift and α is the linear gain of the exponential sine sweep.

## 3. VOLTERRA CONVOLVER

*Volterra Convolver* is the name of the developed software. It is composed by two different tools: an offline Scilab script [5] and a real-time VST plug-in [6]. The former processes the result of the measurement obtaining the leading kernels of the Volterra series basically using (3), the latter implements the formula (2) in discrete time domain: it can be used as a real-time or offline effect in the major VST compatible applications. Adobe Audition 3, Steinberg Cubase SX 3, Cockos Reaper 3.35 have been chosen as reference.

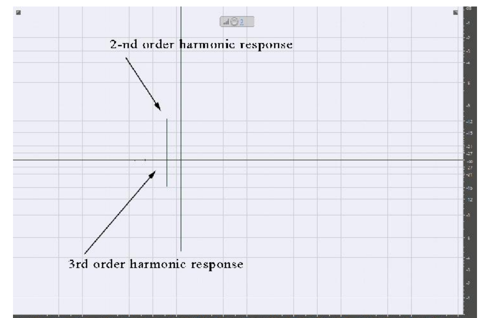



Figure 4: Nonlinear system "train of impulses"

### 3.1 Scilab script and corrective filters

The actual version of the script provides mainly one public (i.e. not for internal use) command:

```
kernelCreator(_rawIrPath, _sineSweepdBGain, _irsLength, _offset,
_irsToIsolate, _T, _f0, _f1, _deltaInvBaseName)
```

The arguments of *kernelCreator(...)* are the full path of the file containing the "train of impulses" (Figure 4) obtained following [3] and [4], the gain (in dBFS) of the sine sweep (ESS) used during the measurement, the length (in samples) of the kernels to obtain, the offset (i.e. the latency of the soundcard), the number of kernels to extract (max 5), the duration of the sweep in seconds, the starting and ending frequencies of the sweep in Hertz. The last parameter permits to load a set of filters developed during the progression of this work that are able to compensate for phase misalignment that mainly appears in the higher order harmonic responses. The development of them has been inspired by the work of Kirkeby, Nelson, Hamada [7] and Farina [8]. The filters depend on the ESS features (length, $f_0$, $f_1$) and afterwards they are applied to $h_2$, $h_3$, $h_4$ and $h_5$ in order to compensate the worsening of the delta Dirac function in the higher order harmonic responses (see Figure 5).

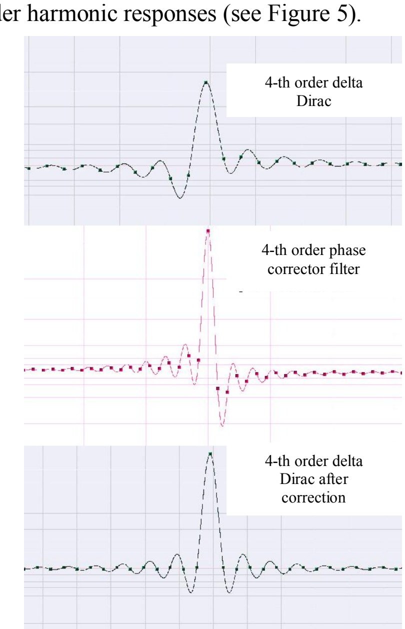



Figure 5: Phase corrector filter effect

### 3.2 VST plug-in

The VST plug-in implements the Volterra series in discrete time domain. The input and output gain can be changed using knobs, the single kernel might be turned on/off and the whole process can be bypassed to permit fast comparisons between "dry" and "wet" signal. By right clicking a kernel switch it is possible to change the output gain of the sound generated by that kernel. Of course presets can be changed using the combo-box. The presets information is stored in an .XML file.


Figure 6: VST plug-in GUI

## 4. EXPERIMENTAL ANALYSES

A basic and clear test was conducted, comparing the results of three different emulations. A guitar overdrive has been measured, its output is the "real output" labelled signal in Figure 8, the following signals are the virtualizations using only the linear impulse response (linear convolution) and using 5 kernels, with and without phase correcting filters. The usefulness of these filters is clear as the last signal in Figure 8 is similar than the third to the reference "real output".

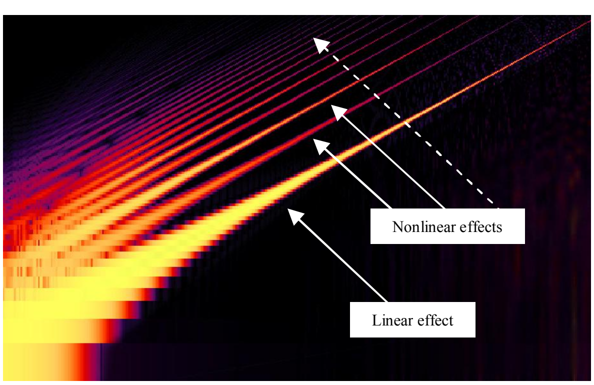



Figure 7: Guitar overdrive spectrograph

A Jupiter C flute has been measured too. The output of the ESS measurement is shown in Figure 9-c. The comparison between spectrographs (Figure 7 and Figure 9-d) reveals the flute has definitely less harmonic distortions than the guitar overdrive.

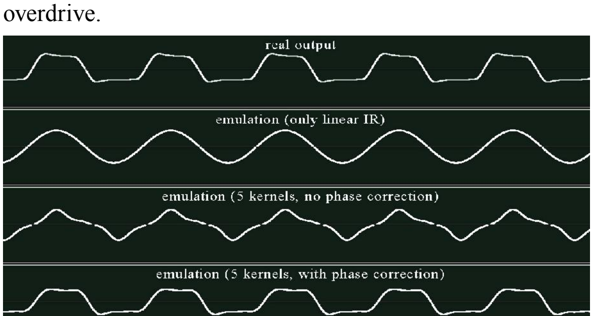



Figure 8: Emulation comparisons

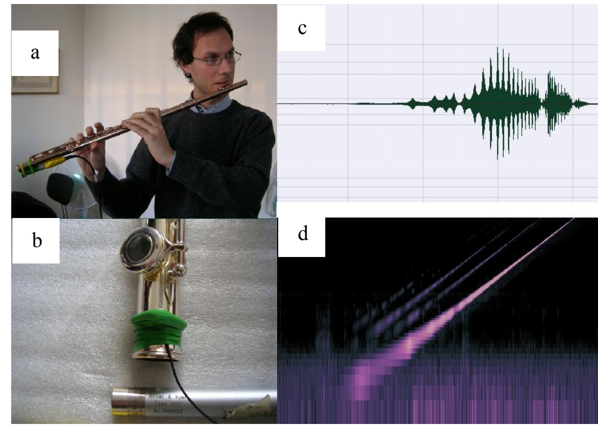
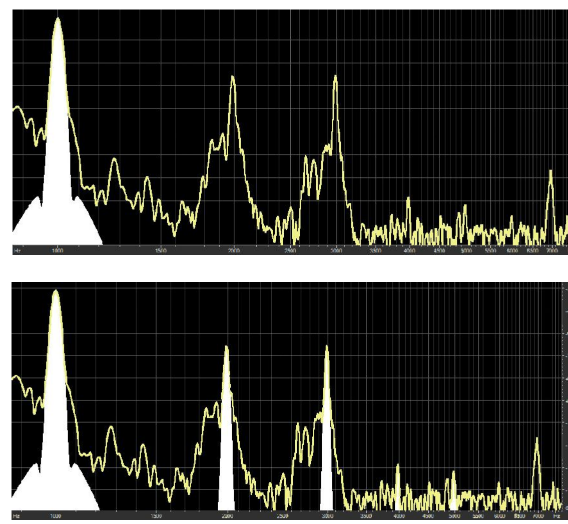



Figure 9: Jupiter C flute measurement session



(a)

(b)

Figure 10: ESS Frequency responses of Jupiter C flute when ESS reaches 1 kHz (yellow line is the real flute frequency response. 10-a is the response using only the linear IR, 10-b is the 5 kernels response)

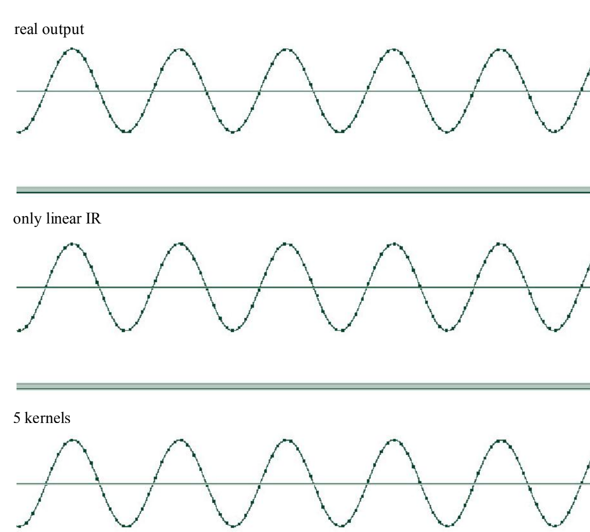



Figure 11: ESS emulation of Jupiter C flute when ESS reaches 1 kHz

After obtaining the kernels an initial comparison between the ESS outputs (i.e. measured and virtualized by the Volterra emulator) has been performed. In a further step, a real performance and a virtualization of the "anechoic" version of that performance has been taken. Nonlinearities of the flute even if well matched by the 5 kernels rendering (Figure 10-b) have little effects on the ESS wave (Figure 11); therefore the emulated performances with and without harmonic distortions will not be much different (Figure 12).


Figure 12: Performances comparison

## 5. CONCLUSIONS

It has been seen that the measurement of harmonic distortion of nonlinear systems with ESS and the following emulation using the diagonal Volterra kernels provides better results than a single IR convolution approach (Figure 8). If the core of the system resides in its low order harmonic distortions the progress of that kind of virtualization is revealed to be evident instead if the system doesn't have sharp harmonic distortions linear convolution is able to provide comparable results (Figure 11 and Figure 12).

As known in nonlinear system harmonics ratios may depend on the volume of the stimulus. To improve the emulation on the whole dynamic of a signal it would be necessary to measure the system with different amplitudes of ESS, collecting the different sets of kernels and based for example on the actual RMS value of a suited length buffered input chose on the fly which set of kernels to use.

## 6. REFERENCES

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