PARAMETER OPTIMISATION FOR WOODWIND SINGLE-REED MODELS

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ABSTRACT

Time-domain modelling of single-reed woodwind instruments usually involves a lumped model of the excitation mechanism. The parameters of this lumped model have to be estimated for use in numerical simulations. Several attempts have been made to estimate these parameters, including observations of the mechanics of isolated reeds, measurements under artificial or real playing conditions and estimations based on numerical simulations. In this study an optimisation routine is presented, that can estimate reed-model parameters, given the pressure and flow signals in the mouthpiece. The method is validated, tested on a series of numerically synthesised data. In order to incorporate the actions of the player in the parameter estimation process, the optimisation routine has to be applied to signals obtained under real playing conditions. The estimated parameters can then be used to resynthesise the pressure and flow signals in the mouthpiece. In the case of measured data, as opposed to numerically synthesised data, special care needs to be taken while modelling the bore of the instrument. In fact, a careful study of various experimental datasets revealed that for resynthesis to work, the bore termination impedance should be known very precisely from theory. An example is given, where the above requirement is satisfied, and the resynthesised signals closely match the original signals generated by the player.

1. INTRODUCTION

Physical modelling of musical instruments aims to translate a set of physical model parameters into audio. In order to synthesise a realistic sound, the value of the model parameters have to be estimated. When modelling single-reed woodwind instruments, special care should be taken regarding the reed excitation mechanism. Two types of non-linearities take place in the mouthpiece. A mechanical one, due to the interaction of the reed with the mouthpiece lay and a fluid dynamical one, due to the complex flow behaviour in the reed channel. In order to incorporate these non-linearities to the physical model, the parameters related to the reed-mouthpiece system have to be estimated accordingly. An overview of experimental and numerical aproaches to the parameter estimation problem is given in Section 2.

Focusing on the clarinet, this paper estimates the parameters of a lumped reed model, based on oscillations generated under real playing conditions (see Figure 1). An inverse modelling process is presented that takes as input the pressure and flow signals in the mouthpiece and estimates the physical model parameters that are required to resynthesise a sound as close as possible to the original sound generated by the player. This process consists of a two-step optimisation routine, outlined in [1].

Section 3 gives a brief description of the lumped reed model that is coupled with a linear resonator model to form the physical model of the instrument and also outlines the formulation of the inverse model, based on a few simplifying assumptions. An example is given, where the inverse model is applied on numerically generated data, in order to validate the accuracy of the presented technique. Section 4 shows how the inverse model performs when applied on measured data and Section 5 discusses the effectiveness of the presented method.

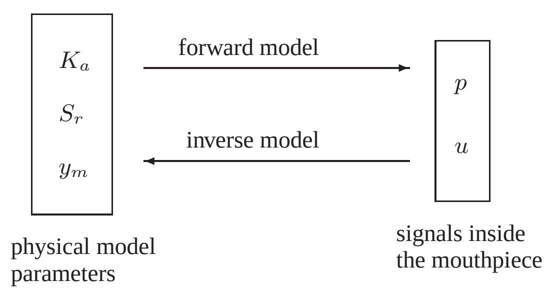

Figure 1: *Forward and inverse modelling of the reed-mouthpiece system, where K_a is the effective stiffness, S_r the effective reed surface, y_m the closing position of the reed and p and u the pressure and flow inside the mouthpiece.*

2. REED PARAMETER ESTIMATION

Considering the simplest possible lumped model of the oscillating reed, namely a single mass-spring model, several attempts have been carried out in order to estimate the parameters that are related with the mechanical and the fluid dynamical properties of the reed-mouthpiece system. In 1969 Nederveen measured the compliance of the reed by observing its deformation in static experiments [2]. Worman [3] managed to experimentally measure the parameters of an isolated reed and his results were adopted by many future authors [4, 5, 6]. Another measurement of the reed stiffness was provided by Gilbert [7] and was in agreement with the compliance measured by Nederveen. These approaches study the mechanics of reeds that are detached from the player, so any actions of the player that affect the system are neglected. In particular humidity might alter the material properties of the reed and it was not considered by the previous authors, besides Worman, who wetted the reed before carrying out his measurements.

The first published attempt to measure reed stiffness under real playing conditions was by Boutillon and Gibiat [8]. Considering a saxophone reed, the authors managed to derive its stiffness by establishing a balance between the reactive powers of the reed and the air-column. The obtained range of values was of the same order with those proposed by Nederveen and Gilbert. Gazengel [9] was the first one to suggest that varying lumped reed model parameters can be estimated using a distributed model of the reed. Using a simplified geometry for both the reed and the mouthpiece, he proposed that estimating non-constant lumped model parameters it would be possible to capture the effect of the reed-lay interaction. In 2003 Dalmont [10] provided experimental results for the internal reed damping and the reed stiffness, pointing out that when near closure a variable stiffness could be assumed in order to capture the non-linear interaction of the reed with the mouthpiece lay. More recently it has been shown how using a distributed model of the reed that captures the interaction with the lips and the mouthpiece lay, it is possible to estimate lumped reed model parameters, while considering a quite realistic geometry of the system [11, 12].

Concerning the flow in the mouthpiece, Backus [13, 14] used an artificial blowing method to obtain measurements for the vibrations of the clarinet in order to derive an expression between the volume flow and the reed opening. Nederveen also presented findings that were in qualitative agreement with those of Backus. A more extensive study on the fluid dynamics of the system along with experimental results [15, 16, 17, 18] observed the fact that the flow separates from the walls of the reed channel, forming an air jet. This fact was confirmed by numerical simulations of the flow in the mouthpiece [19] and incorporated in a refined reed model [20].

3. A LUMPED SINGLE-REED MODEL

3.1. The forward model

The equation of motion for a single mass-spring model of the reed is

$$m\frac{d^2y}{dt^2} + mg\frac{dy}{dt} + K_a(\Delta p)y = \Delta p, \tag{1}$$

where y is the reed displacement, m the mass per unit area and g the damping per unit area. The effective stiffness per unit area, K_a, is treated as a function of Δp, the pressure difference across the reed, thus rendering the model able to incorporate the quasi-static mechanical non-linear behaviour of the system.

The formation of the air jet in the reed channel can be taken into account by introducing an effective reed width that can scale down the opening surface (here assumed to be rectangular). Thus, the "vena contracta" effect [16, 20] is considered, assuming that:

$$\alpha S_f \approx \lambda h, \tag{2}$$

where α is the vena contracta factor, S_f the opening surface, λ the effective width of the reed and h the reed opening. The flow inside the reed channel (u_f) can be derived by Bernoulli's equation for ideal fluid flow and is expressed as

$$u_f = \lambda h\sqrt{\frac{2\Delta p}{\rho}} \tag{3}$$

where ρ is the air density. The flow induced by the oscillation of the reed is given by

$$u_r = \frac{dy}{dt}S_r, \tag{4}$$

with S_r the effective moving surface of the reed.

The mouthpiece pressure p can be decomposed into a wave going into (p^+) and out (p^-) of the bore, which are related to the total volume flow $u = u_r + u_f$ by

$$Z_0 u = p^+ - p^-, \tag{5}$$

where Z_0 is the characteristic impedance at the mouthpiece entry. Combining equations (3) and (5) yields the non-linear equation for u_f

$$\text{sign}(u_f)\frac{\rho}{2(\lambda h)^2}u_f^2 + Z_0 u_f + (2p^- - p_m + Z_0 u_r) = 0, \tag{6}$$

where p_m is the blowing pressure. The above lumped element can be coupled to a digital bore model, to create a feedback loop that completes the digital representation of the instrument (see [21, chapter 4]) for details).

3.2. The inverse model

The first step towards the parameter optimisation is based on the simplifying assumption that the the reed displacement y is proportional to the pressure difference Δp across it [22]:

$$y = C\Delta p = C(p_m - p), \tag{7}$$

where C is the compliance of the reed [2] and p the pressure inside the mouthpiece. The reed opening h can be related to y as

$$h = y_m - y, \tag{8}$$

with y_m the closing position of the reed. Under this assumption the total flow into the mouthpiece as a function of the reed displacement y is

$$\begin{aligned} u &= u_f + u_r \\ &= (-\lambda y + \lambda y_m)\sqrt{\frac{2(p_m - p)}{\rho}} - CS_r\frac{dp}{dt} \\ &= c_1\sigma\sqrt{\frac{2}{\rho}}(p_m - p)^{3/2} + c_2\sigma\sqrt{\frac{2}{\rho}}(p_m - p)^{1/2} + c_3\frac{dp}{dt}, \end{aligned} \tag{9}$$

where σ is the sign of $(p_m - p)$ and

$$\left\{\begin{array}{l} c_1 = -C\lambda \\ c_2 = y_m\lambda \\ c_3 = -CS_r \end{array}\right. \Rightarrow \left\{\begin{array}{l} K_a = -\lambda/c_1 \\ y_m = c_2/\lambda \\ S_r = \lambda c_3/c_1 \end{array}\right. .$$

Assuming that the pressure and flow signals in the mouthpiece are known, and taking as objective function the mean square error between the original flow signal and the estimated flow as calculated from equation (9), it is possible to use the Nelder-Mead optimisation algorithm [23], in order to get a first estimate for K_a (constant), S_r, y_m and p_m [22]. These estimated parameters enable the synthesis of oscillatory signals; they lie within a range so that the simulation of sustained clarinet notes is possible. It remains to fine-tune them so that we get a better match between the original and the resynthesised signals.

For the fine-tuning process (second optimisation step) we use as objective function the mean square error of the pressure signals at the steady state:

$$f_{obj} = ||p_{or} - p_{res}|| \,. \tag{10}$$

where p_{or} is the original and p_{res} the resynthesised pressure in the mouthpiece. This non-linear optimisation problem is solved using the Rosenbrock method [24]. In contrast to the first optimisation step, it is now possible to include in the model all the physical parameters that govern the oscillations of the system, namely K_a, S_r, y_m, p_m, effective mass m, damping g and effective width λ. Furthermore, at this stage the effective stiffness per unit area (K_a) is estimated as a function of the pressure difference along the reed, being constant when there is no reed-lay interaction, and rising linearly for higher Δp values,

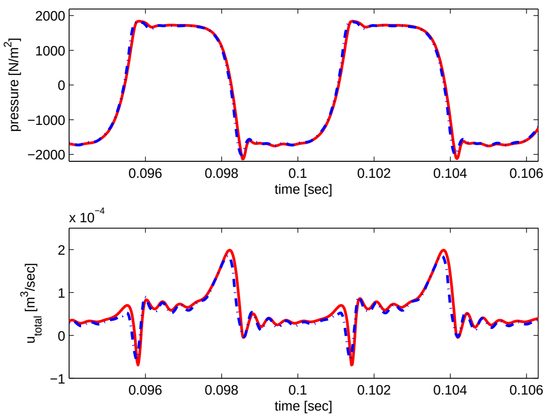

Figure 2: Pressure (top) and flow (bottom) signals in the mouthpiece for the numerically synthesised sound (dotted-blue) and the resynthesised sound using the estimated parameters (red).

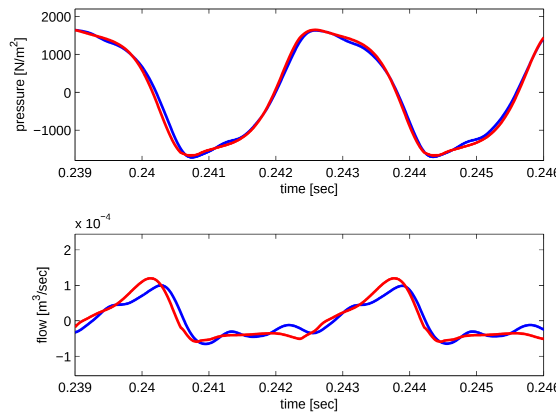

Figure 4: Pressure and flow signals in the mouthpiece for the measured (blue) and the resynthesised sound (red).

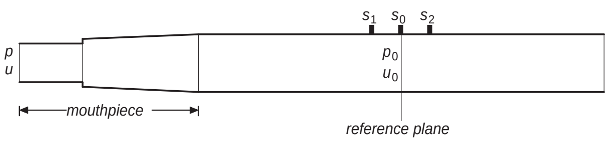

Figure 3: Schematic depiction of the experimental set-up.

when interaction takes place [1]. The resynthesised pressure signal should, at every iteration of the Rosenbrock algorithm, lie closer to the pressure signal that was used as an input. Thus by starting with two signals that lie reasonably close to each other, something achieved during the first optimisation step, it is possible to reach a suitable set of parameters that produces almost identical results.

Applying the whole two-step routine on numerically synthesised signals, and since the same model is used to create and resynthesise the signals, an almost perfect match is required in order to validate the inverse modelling approach. One example comparison between the input and the resulting pressure and flow in the mouthpiece is depicted in Figure 2. The values of the estimated parameters, after both optimisation steps, as well as the parameters used to create the original, input signals are listed on Table 1, where the effectiveness of the second, fine-tuning step is made apparent.

4. APPLICATION TO MEASURED DATA

The measured data is obtained from experiments with blowing a simplified clarinet, the schematic bore profile of which is shown

	theoretical	estimated (I)	estimated (II)	units
K_a	$8.66 \cdot 10^6$	$8.97 \cdot 10^6$	$8.67 \cdot 10^6$	N/m^3
S_r	$7.61 \cdot 10^5$	$8.42 \cdot 10^5$	$8.33 \cdot 10^5$	m^2
y_m	$4 \cdot 10^{-4}$	$3.2 \cdot 10^{-4}$	$3.68 \cdot 10^{-4}$	m
p_m	1800	1919	1825	Pa
λ	0.013	—	0.0142	m
m	0.05	—	0.044	Kg/m^2
g	3000	—	3805	1/sec

Table 1: Theoretical vs. estimated parameters.

in Figure 3. Travelling pressure waves are calculated at the reference plane (p_0, u_0) and translated to pressure and flow signals in the mouthpiece. More details about the above process and concerning the model of the resonator of the instrument can be found in [25, 26, 1], as well as in [21, chapter 6].

The two-step optimisation routine is applied on a slice of the data that resembles the steady state of the measured sound. In order to obtain a meaningful estimation of the reed model parameters it is necessary to use an accurate model of the instrument bore. The complex geometry of the clarinet mouthpiece, as well as the radiation impedance of the cylindrical tube influence the input impedance of the resonator, thus affecting the parameter optimisation. Using a baffle termination of the cylindrical pipe allows for a more accurate representation of the termination impedance [27], resulting in a good match between theoretical and experimental input impedance, as depicted in Figure 5. After carrying out the optimisation routine, the estimated reed model parameters can be used to resynthesise the pressure and flow signals in the mouthpiece, as seen in Figure 4.

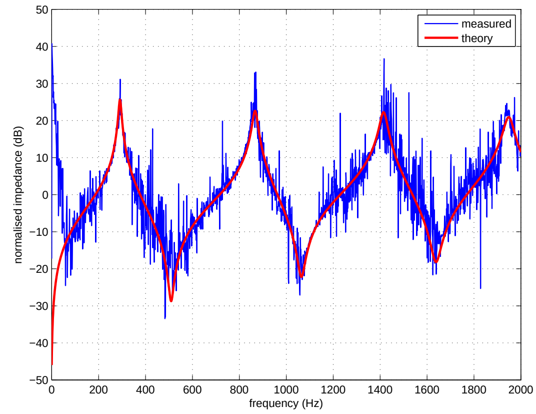

Figure 5: Theoretical and experimental input impedance of the flanged pipe used for the experiments.

5. CONCLUSIONS

Estimating the reed model parameters from signals generated by a real player requires a model of the instrument that can adapt to the properties of the experimental set-up. Concerning the bore model, errors can arise during the acquisition of the signals at the reference plane, and their translation to the mouthpiece. Such errors can cause a mismatch between the input impedance of the model (obtained from theory) and that of the simplified clarinet used in the experiments. Taking special care in order to ensure that

$$p^- = r_f * p^+ \tag{11}$$

where r_f is the reflection function of the tube, turns out to be a pre-requisite for a successful resynthesis. Remaining deviations from the original signals observed in Figure 4, especially in the case of the flow signal, are caused by uncertainties in the reed model, due to unpredictable fluid dynamical phenomena [19, 20] and a more complex reed-lay interaction than that predicted by the physical model.

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