Here we can see that the influence of the edge curvature on the string velocity and on the string displacement is similar. In vicinity of the edge the string velocity is equal almost to zero along a distance, which becomes longer with increasing of the radius of curvature of the edge.

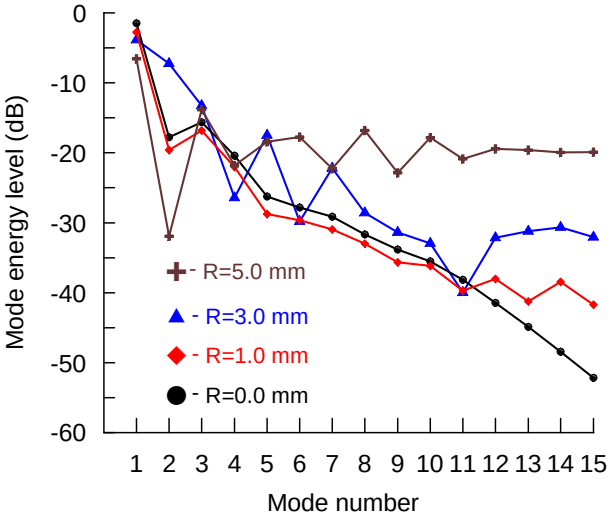

Figure 5: Spectral envelopes for string $n = 70$. Varying the edge curvature R with fixed hammer velocity $V = 3$ m/s.

In Figure 5 we demonstrate the influence of the edge curvature on the spectrum of the string vibrations excited by the hammer with initial velocity is $V = 3$ m/s.
It is clear that with increasing of the edge curvature the amplitude of higher harmonics becomes greater. Moreover, the form of the spectral envelopes for $R = 3$ mm and 5 mm is essentially irregular, and the rate of higher harmonics attenuation changes significantly. Obviously, the edge curvature $R \geqslant 5$ mm creates the train of oscillations up to very high frequencies. Finally, we can see the strong influences of the edge curvature on the amplitude of the second harmonic, and this fact is very important.

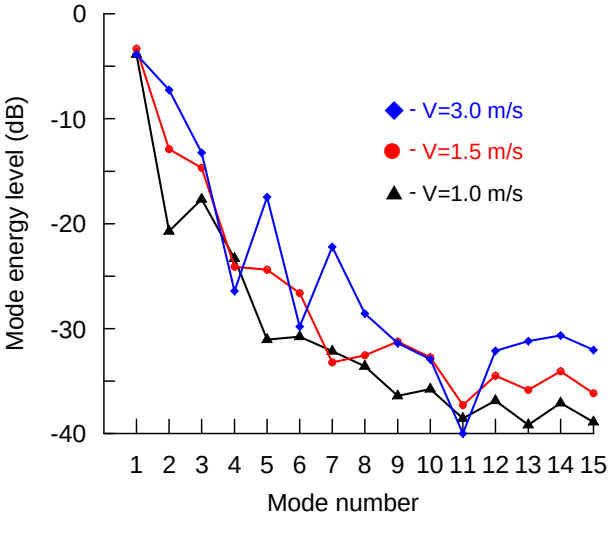

Figure 6: Spectral envelopes for string $n = 70$. Varying hammer velocity V with fixed edge curvature $R = 3$ mm.

In Figure 6 is presented the influence of the velocity of the hammer strike on the spectrum of the string vibrations.
It is evident that the power spectrum of the string vibration grows up significantly and reshapes essentially with increasing of the hammer velocity.

6. CONCLUSIONS

We have provided a careful model of piano string with nonlinear support, and found that this theory may be of some use for piano treble strings, which one end is terminated on the curved edge of the frame. One respect in which this model is still idealized is its assumption about a very simple string boundary condition at the piano bridge.
It is found that the new trains of high frequency oscillations that do not exist initially grow up eventually, and its appearance depends on the curvature of the edge of the frame. It is shown that the power spectrum of the string vibration is enriched by spectral components up to very large numbers, and essentially reshapes with increasing of the amplitude of the initial wave exited by the piano hammer.
It is revealed that even the small variation of the edge curvature significantly influenced on the amplitude of the second harmonic in fact. For this reason the manufacturers of grand pianos should produce a cast iron frame very accurately, and carefully process the surface of the edge.

7. ACKNOWLEDGEMENT

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